AFM-Notes 5.1

Evaluate (with a calculator) each of the following exponential functions. Round to <u>four</u> decimal places if necessary.

1. $f(x) = 340(0.2)^x$ $x = 4$
2. $g(x) = 0.6(3)^x$ $x = 6$

Recursive formula

a. u_0

b. u_n=(number with exponent)u_{n-1}

Explicit function

a. $y = u_0$(then number in front of $u_{n-1})^x$

Find the next three terms for each sequence. Then write an explicit function for the sequence.

1. $u_0 = 48$
   $u_n = 1.2u_{n-1}$
2. $u_0 = 120$
   $u_n = 0.5u_{n-1}$

Evaluate each function at x = 0, x = 1, and x = 2. Then write a recursive formula for the pattern.

1. $f(x) = 4(2)^x$
2. $f(x) = 2400(0.25)^x$

Growth function: number inside ( )'s greater than 1

Decay function: number inside ( )'s greater than 0 but less than 1

Determine whether each equation is a model for exponential growth or decay

1. $f(x) = 1700(1 - .42)^x$
2. $g(x) = \frac{4}{5}(1+0.5)^x$

Calculate the ratio of the second term to the first term, and express the answer as a decimal value. State the percent increase or decrease.

1. 600, 120
2. 64, 115.2
3. 28, 98

Sal bought a new boat for $36,200. The value of the boat is depreciating at a rate of 12% a year.

a. Write a recursive formula that models this situation. u_0 represents the purchase price.
b. Find the value of the boat after 1 year, 4 years and 6 years.
c. Write an exponential equation that models this situation.

AFM-Notes 5.2

Rewrite each expression as a fraction without exponents. Verify using a calculator.
*Negative exponent in the numerator becomes a positive exponent in the denominator.
*Negative exponent in the denominator becomes a positive exponent in the numerator.

1. 5^{-3} 2. -2^{-6} 3. $(-2)^{-6}$ 4. $\left(\frac{2}{3}\right)^{-3}$ 5. $-\left(\frac{4}{5}\right)^{-2}$ 6. $\left(-\frac{7}{6}\right)^{-4}$

Rules of exponents:
1. Multiply with the same base: Add exponents
2. Divide with the same base: Subtract exponents (numerator exponent – denominator exponent)
3. Exponent on the outside of ( )'s: Simplify inside the ( )'s if possible, then distribute outside exponent over All exponents inside.
4. Anything to the zero power = 1 (except 0^0 is undefined)
5. No negative exponents left in the answer: See above

Rewrite each expression in the form ax^n or x^n.

1. $x^{-8}\bullet x^3$ 2. $(-18x^{-7})(3x^{-2})$ 3. $\frac{-24x^{-2}}{8x^{-12}}$ 4. $\left(\frac{-54x^5}{9x^{11}}\right)^{-2}$

5. $\left(\frac{15x^0}{12x^{-2}}\right)^{-3}$

Solve when x is an exponent:
1. break down both sides of = to get the same base (Use rules of exponents)
2. drop the base - Solve

Solve for x:

1. $3^x = \frac{1}{729}$ 2. $256^x = 64$ 3. $\left(\frac{32}{9}\right)^x = \frac{243}{128}$

Solve equations when x is the base.
1. Isolate the variable
2. Take the root of each side if exponent is an integer: Raise both sides to a power is fractional exponent.

Solve, is not exact answers, round to two decimal places.

1. $x^7 = 2893$ 2. $x^{0.6} = 56$ 3. $4x^{1.5} = 90$ 4. $16x^8 = 12x^5$ 5. $600x^{-4}=225x^{-6}$

AFM Notes 5.3-Rational exponents and Roots

Power function: Only one x term with a number as an exponent. $y = ax^n$ a and n are numbers.
Exponential function: Has a variable as an exponent. $y = ab^x$ where a is the y-intercept and b is the growth rate.

Identify the following as a power function, an exponential function or neither.

1. $f(x) = 32(0.65)^x$
2. $g(x) = x^3 - 7x^2 + 8x - 20$
3. $h(x) = \frac{5}{x^2} - 9$

Rational exponents (fractions) $\sqrt[index]{base^{exponent}}$

1. The index is the denominator
2. The exponent is the numerator.

Rewrite using rational exponents.

1. $\sqrt[6]{b^2}$
2. $\sqrt{a^5}$
3. $\left(\sqrt[4]{d^3}\right)^2$
4. $\frac{1}{\sqrt{r^7}}$
5. $\frac{1}{\sqrt[7]{c^2}}$

1. isolate the radical
2. Write radical using rational exponents
3. Raise **BOTH** sides to the reciprocal from step #2

Solve each equation. If answers are not exact, approximate them to the nearest hundredth.

1. $\sqrt{4x^5} + 24 = 30$
2. $\frac{1}{\sqrt[3]{x}} = 0.21$
3. $6\sqrt[4]{x} + 20 = 23$
4. $\sqrt[5]{x^4} - 19 = -15$
5. $\sqrt{27x^3} = 40$

Notes 5.4 Applications of Exponential and Power Equations

(A) $168.4 = 4x^{1.5}$

(B) $500 = 120(1+\frac{x}{6})^7$

% increase / decrease

$$\text{equation} = \frac{\text{difference between}}{\text{original amount}}$$

The price of a book increased from \$35 to \$62.40 over 4 years.
Find the average yearly % increase.

The population of birds has been increased. In 1998 there were 46,000 and 2001 there were 72,000

a) write the equation:

b) How many birds do you predict for 2008:

2018:

3018:

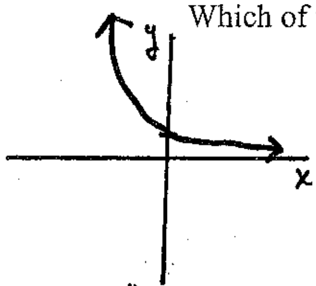
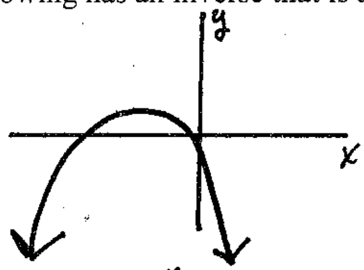
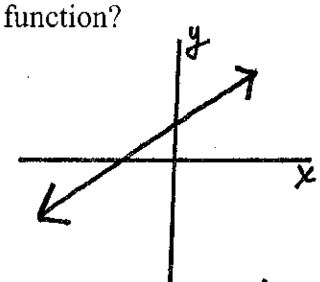
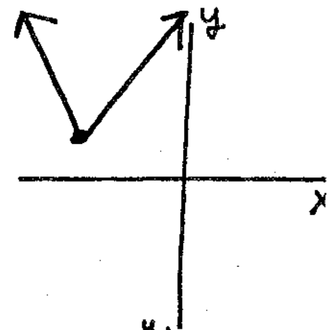
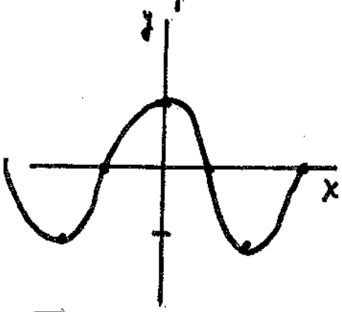
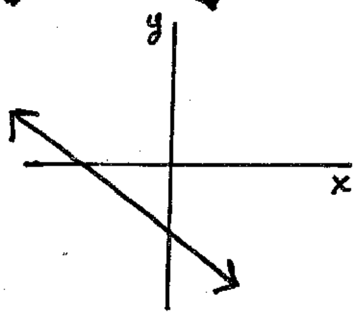
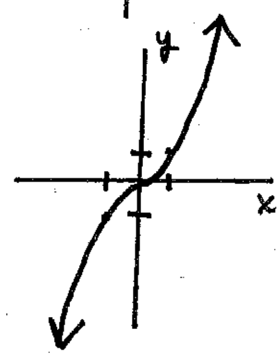
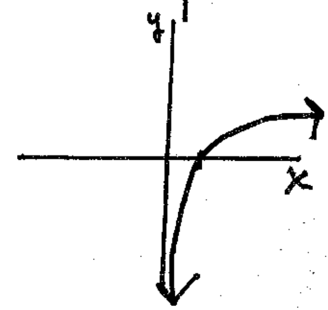
AFM-Notes 5.5 Inverses

Graph:

f(x) is a function if it passes the ________________ line test.

f^{-1} (the inverse of f(x)) is also a function if **f(x)** passes the ________________ line test.

Which of the following has an inverse that is also a function?

If f(x) is a function that f^{-1} is also a function then f(x) is said to be a **ONE-TO-ONE FUNCTION.**

Domain of f(x) is the range of f^{-1}

Range of f(x) is the domain of f^{-1}

The following functions have an inverse that is also a function. Find four points on the graph of each function f, using the given values of x. Use those points to identify four points on the graph of f^{-1}.

1. $f(x) = -3x + 7$; $x = -4, 0, \frac{8}{3}, 5$

2. $f(x) = 2^x$; $x = -2, 0, 3, 9$

Evaluating functions and functions inverses. SUBSTITUTE AND SOLVE

Given $f(x) = -x^3 + 2$, find

1. f(3)   2. f(-5)   3. 3. $f^{-1}(-6)$ 4. $f^{-1}(44.875)$

To determine if a function has an inverse that is also a function:

1. graph f(x)
2. perform a horizontal line test on f(x)

To find the equation of a functions inverse.

1. replace x with y
2. replace f(x) or y with x
3. solve for y

For each function, determine whether or not the inverse is also a function. Find the equation of the inverse and graph both on the same axes.

1. $y = 5x - 2$ 2. $y = (x+2)^2 - 3$ 3. $y = \sqrt{4 - x^2}$ 4. $y = x^4$

AFM notes 5.6 Logarithmic Functions

Exponential Function: $y = a^x$ Logarithmic Function: $\log_a y = x$

If there isn't an "a" number it is understood to be 10.

THEY ARE INVERSES

Write an equation of the inverse of each function.

1. $g(x) = 7^{2x}$
2. $f(x) = \log_3 x$
3. $g(x) = \log 16$

Rewrite in exponential form and solve for x.

1. $\log x = 3$
2. $\log_3 \frac{1}{729} = x$
3. $\log_5 1 = x$
4. $\log_6 \sqrt[3]{6^2} = x$
5. $x = \log_8 128$
6. $\log_7 \frac{1}{2401} = x$

To find the <u>exact</u> value without a calculator

1. Rewrite in exponential form: Use x as the exponent
2. Get the same base: Use rules of exponents
3. Drop base and solve

Find the exact value without a calculator.

1. $\log .0001$
2. $\log_5 \frac{1}{625}$
3. $\log_2 \sqrt[3]{16}$
4. $\log_3 \sqrt{27}$

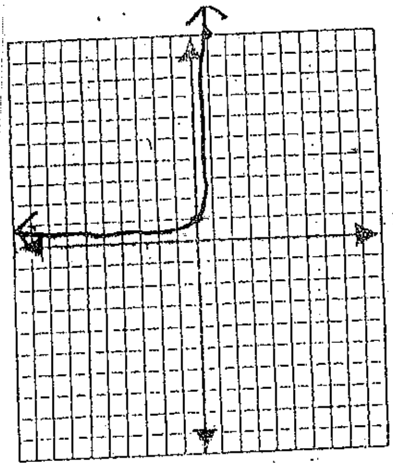
Transformations: Exponential functions pass through (0, 1)
Logarithmic functions pass through (1, 0)

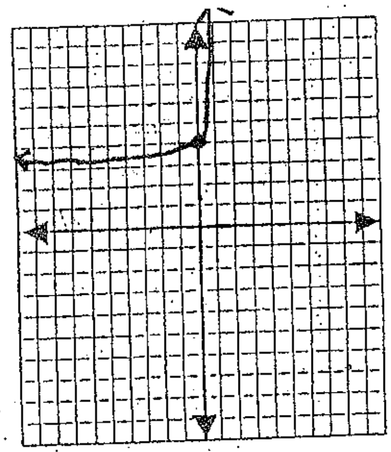
Change of base: Use if you cannot get the same base.

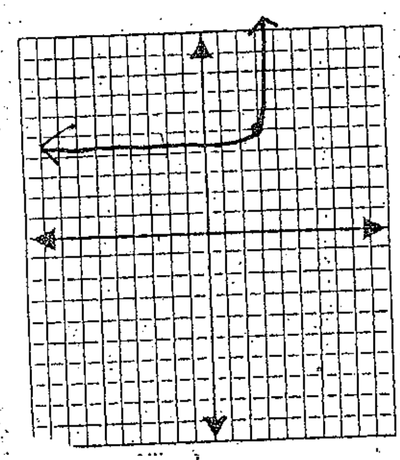
1. rewrite exponential functions as log
2. $\frac{\log a}{\log y}$ Round to four decimal places

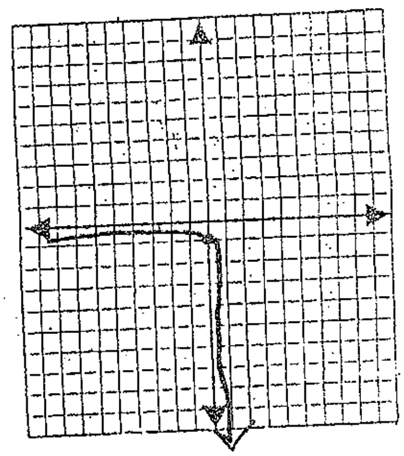
1. $\log_3 2589$
2. $6^x = .0126$
3. $15^x = 60$
4. $\log 14{,}524$

$y = 10^x$

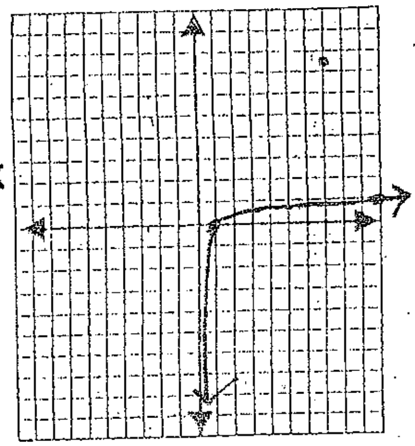


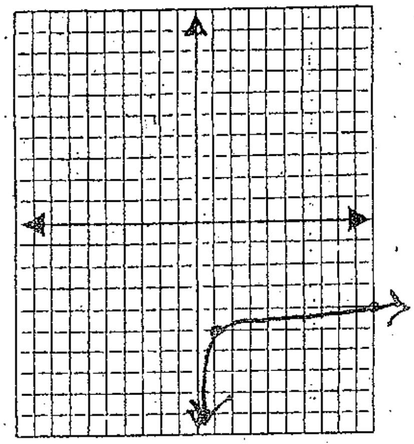


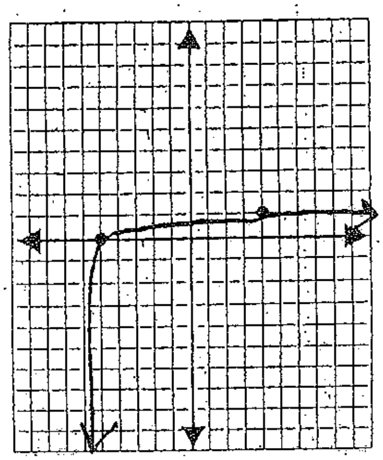


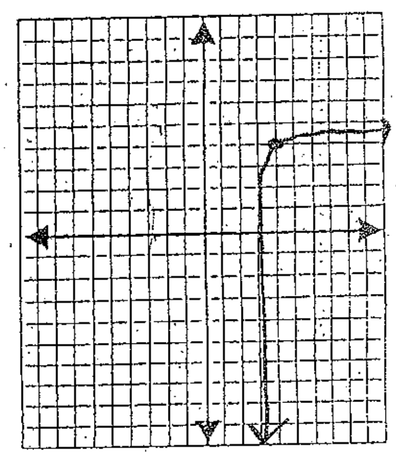


y=logX









AFM Notes 5.7 Properties of Logarithms

Remember that a logarithm is the inverse of an exponential.

1. $\log_a M*N = \log_a M + \log_a N$ (exp. Multiply with the same base add exponents)
2. $\log_a M^k = k\log_a M$ (exp. Power to power multiply inside with outside exp)
3. $\log_a \frac{M}{N} = \log_a M - \log_a N$ (exp. Divide with the same base subtract exps top – bottom)
4. $\log_a S = \frac{\log S}{\log a}$ Change of base

Use properties of logs or exps to change the form of the following:

1. $4\log_a D$
2. r^{x+y}
3. $\log_a G + \log_a L$
4. $(3x^2y^8)^4$
5. $\frac{\log_b P}{\log_b Q}$
6. T^{a-b}
7. $\log_c R$
8. $\log_a W + \log_a X - n\log_a Y$

Determine if the following are true or false.

1. $\log 28 = \log 7 + \log 4$
2. $\log 15 = \frac{\log 45}{\log 3}$
3. $\log \sqrt[3]{12} = -3\log 12$

Use properties of logarithms to expand the following.

1. $\log xy^5z^3$
2. $\log_3 \frac{ab^2}{c^4}$
3. $\log_5 d^7$

To solve exponential equations as x as an exponent:

1. isolate the term with the exponent
2. Take the log of BOTH sides of =
3. Use property #2 from above to write as a multiplication problem
4. Divide to get both logs on the same side of the = (change of base)
5. Divide again if there is a number in front of x (Round to 4 decimal places)

Solve:

1. $87 + 4.3^x = 115$
2. $24(0.35)^{3x} = 500$
3. $92 + 8(2.5)^{5x} = 1812$